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### FORMULATION OF VLASOV'S ENERGY THEOREM FOR TIMBER BOX BEAMS

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#### ABSTRACT

This work applied Vlasov's theory and principle of minimum potential energy to obtain the equation of equilibrium of timber box beam made of anisotropic material (timber). This equation is similar to that of beam on elastic foundation (BEF). The closed form solution of the obtained equation enabled distortional warping and pure torsional stresses of anisotropic timber box beam to be evaluated. A comparative analysis of the theoretical results (results obtained by analysis of timber box beams using the development here) and the experimental results of the same box beams was carried out and there is rapor between both results even when anisotropic timber of different species box beams of uniform web- flange thickness and variation of web- flange thicknesses were used.

**KEYWORDS:** Beam, Box, Timber, Torsion, Vlasov.

#### INTRODUCTION

The strength of a structure is a function of the strength of the material used in constructing it. As long as the materials strength is designed to the strength of the prevailing condition the structure will not fail. When someone wants to design a material it is very necessary to consider the strength of the material, therefore the strength of the material is a governing factor. Considering the well known engineering materials like timber, concrete, steel, ceramic and plastics, some of these materials has certain induced improvements like reinforced concrete, prestressed concrete, plywood laminated veneer lumber, and structural timber composites. This shows that every material has intrinsic variations in properties. In engineering, it is necessary to understand the variations in certain material by looking at different parameters as in its properties. This will help engineer to be fully knowledgeable on reasons below. Why do trees fall? Why do buildings fall? Why do structures fall? Why do things fall?

#### STATEMENT OF THE PROBLEM/RESEARCH ISSUES:

The focus of this study is defined by the problem in the analysis and design of timber box beams. The need to obtain an optimization technique that can be formulated from Vlasov's concept. This will be useful and relevant in cost optimization of timber construction works and other engineering construction. The problem of research output in the application of connection/fastening techniques, and timber bridges construction as in box bridges, box beams and roof beams can be tackled. The research solves the problem of creating accessibility in river line areas and water logged zones in Nigeria by creating an alternative bridge design and fabrication mechanism whether temporal or permanent that is cheaper, affordable and available. Also new understanding of the torsional response of the timber material used in different fabrication of structural members has to be created. So that long spans beams can equally be fabricated in timber.

#### AIMS OF THE WORK:

- To formulate an analytical model for assessment of the torsional response of timber as anisotropic material, consequently to develop a minimum potential energy equation that can be minimized to create stability of beam systems.

#### OBJECTIVES OF THE WORK:

The main objectives of this research work is to provide an alternative mechanism and material that can bring about the following

- i) Cost optimization of timber trusses and beams.
- ii) Cost optimization of timber bridges as in box bridges.
- iii) To analysis numerically the stresses in cells of box bridges and the connection /fastening techniques.
- iv) Using existing Vlasov's equation with slight modification to analysis and design timber box beams problems.

#### LITERATURE REVIEW:

Borg and Gennaro as in Ezeagu (2008) summarily stated that "Basically there are three different procedures for determine the deflection of engineering structures: They include:

(1)Integration of the deflection differential equation of the beam. (2) The strain energy stored in a structure and the use of the law of conservation of energy. (3) Graphical methods. Vafai A. and Pinucs G (1973); Zakie D. B;(1973) Ramos A. N; (1961) Brian S; (2005). Timoshenko S. P; ( 1936,1953), Sokolnikoff IS; 1946 ,Westerguard H, M;(1942). Bhatt P and Nelson H. M; (1994 Osadebe N.N and Mbajjogu M.S.W;(2005). Saada F.S; Heins C.P;(1975). Rekach V.G; Elsgolts L.

**METHODOLOGY:**

The general method adopted in this research is as follows:  
Formulation of a valid equation (Vlasov’s energy theorem for anisotropic materials). This will involve the development of mathematical model, an analytical model (formulation of Vlasov’s theorem for analysis and design of timber box beams).

To optimized the formulated minimum energy equation of Vlasov’s theorem for box beams in order to create a stable system and compare the analysis of experimental values with the analytical value.

**ANALYTICAL FORMULATION OF ENERGY EQUATION FOR TIMBER BOX BEAM (ANISOTROPIC MATERIAL).**

Timber possesses three perpendicular planes of elastic symmetry i.e.,

- i) Longitudinal properties (properties in X direction)
- ii) Transverse properties (properties in Y direction) or S co-ordinate or Tangential.
- iii) Radial properties (properties in Z direction which is assumed negligible)

The corresponding deformations are as noted.

- i) the longitudinal displacements is warping (U)
- ii) the transverse displacements is distortional (V).

The corresponding known functions are:

- U (x,s)----- longitudinal displacements in x direction
- U (y,s)-----longitudinal displacement in y direction.
- V (x,s)-----transverse distortional displacement in x direction.
- V (y,s)-----transverse distortional displacement in y direction

The general equation is in the form of the relationship

$$E = 2 G (1 + \gamma) \text{----- (4.1)}$$

Where

E = modulus of Elasticity; G = Modulus of Rigidity;  
γ = The directional and material property – Poisson’s ratio.  
The stress –strain relationships are as follows

$$\epsilon_x = \frac{\sigma_x}{E_x} - \gamma \frac{\sigma_y}{E_y} + \alpha T \text{----- (4.2)}$$

$$\epsilon_y = \frac{\sigma_y}{E_y} - \gamma \frac{\sigma_x}{E_x} + \alpha T \text{----- (4.3)}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \text{----- (4.4)}$$

Where

E<sub>x</sub> = Young’s modulus in x – direction; E<sub>y</sub> = young’s modulus in y – direction  
γ<sub>x</sub> = Poisson’s ratio in x – direction; γ<sub>y</sub> = Poisson’s ratio in y direction

Ignoring the temperature component of the stress-strain equation,  
Equation (4.2) becomes

$$\epsilon_x = \frac{\sigma_x}{E_x} - \frac{\gamma\sigma_y}{E_y} \text{-----} (4.5)$$

and equation (4.3) becomes

$$\epsilon_y = \frac{\sigma_y}{E_y} - \frac{\gamma\sigma_x}{E_x} \text{-----} (4.6)$$

To obtain an expression of direct stress in the principal axis, making  $\sigma_x$  subject of formula from equation (4.5)

$$\therefore \sigma_x = E_x \epsilon_x + \gamma \frac{E_x}{E_y} \sigma_y \text{-----} (4.7)$$

Also from equation (4.6)

$$\therefore \sigma_y = E_y \epsilon_y + \gamma \frac{E_y}{E_x} \sigma_x \text{-----} (4.8)$$

Substituting equation (4.8) in equation (4.7)  
eqn (7) becomes

$$\sigma_x = E_x \epsilon_x + \gamma \frac{E_x}{E_y} \left[ E_y \epsilon_y + \gamma \frac{E_y}{E_x} \sigma_x \right] \text{-----} (4.9)$$

multiplying out equatio (4.9) becomes

$$\sigma_x = E_x \epsilon_x + \gamma E_x \epsilon_y + \gamma^2 \sigma_x \text{-----} (4.10)$$

Collecting like terms.

$$\sigma_x - \gamma^2 \sigma_x = E_x \epsilon_x + \gamma E_x \epsilon_y \text{-----} (4.11)$$

Simplifying further,

$$\sigma_x [1 - \gamma^2] = E_x [\epsilon_x + \gamma \epsilon_y] \text{-----} (4.12)$$

$$\sigma_x = E_x \frac{[\epsilon_x + \gamma \epsilon_y]}{1 - \gamma^2} \text{-----} (4.13)$$

Similarly, equation of direct stress in y direction becomes,

$$\sigma_y = \frac{E_y [\epsilon_y + \gamma \epsilon_x]}{1 - \gamma^2} \text{-----} (4.14)$$

Equation (4.13) and (4.14) represents the two principal stresses equivalent system that can be generated from equation (4.7) and (4.8)

In summary,

$$\sigma_x = E_x \frac{[\epsilon_x + \gamma \epsilon_y]}{1 - \gamma^2} \text{-----} (4.13)$$

$$\sigma_y = E_y \frac{[\epsilon_x + \gamma \epsilon_y]}{1 - \gamma_2} \dots\dots\dots (4.14)$$

And

$$\tau_{xy} = Y_{xy} G \dots\dots\dots (4.15)$$

Using the above displacement fields and basic strain relationships of the theory of elasticity, the expressions for normal stress and shear becomes.

$$\sigma_{(x,s)} = E_x \left[ \frac{\partial u(x,s)}{\partial x} \right] + \left[ \frac{\partial v(x,s)}{\partial s} \right] = E_x \left[ \frac{du(x)}{dx} \phi(s) \right] + E_x \left[ \frac{dv(s)}{ds} \phi(s) \right] =$$

$$E_x \left( u'_{(x)} \phi(s) + V'_{(x)} \phi(s) \right) \dots\dots\dots (4.15)$$

$$\tau_{(x,s)} = G \left[ \frac{\partial u(x,s)}{\partial s} + \frac{\partial v(x,s)}{\partial x} \right] = G \left[ U_{(x)} \frac{\partial \phi(s)}{\partial s} + \frac{\partial V(x)}{\partial x} \psi(s) \right] =$$

$$G \left[ U_{(x)} \phi'_{(s)} + V'_{(x)} \psi(s) \right] \dots\dots\dots (4.16).$$

In a unit cell of a thin walled box structure under generalized loading,  
See fig. 4.1 and fig. 4.2

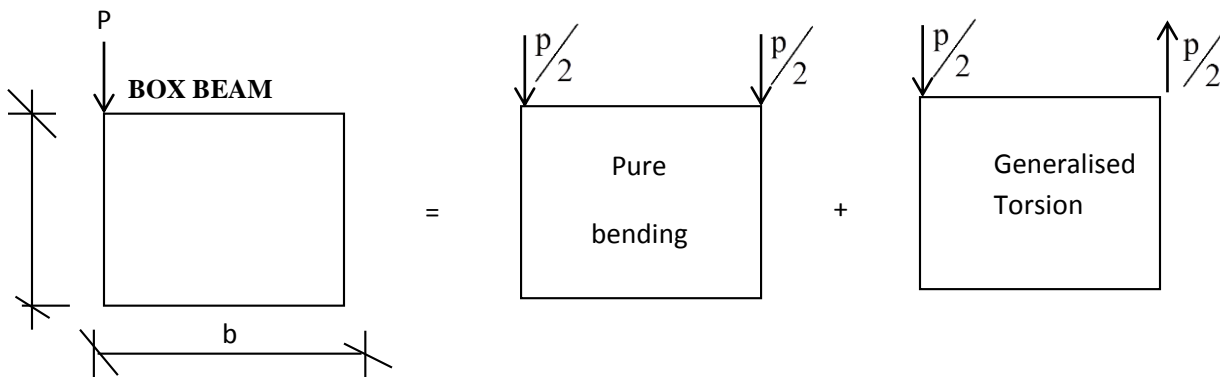


Fig.4.1: Box beam under generalized loading condition.

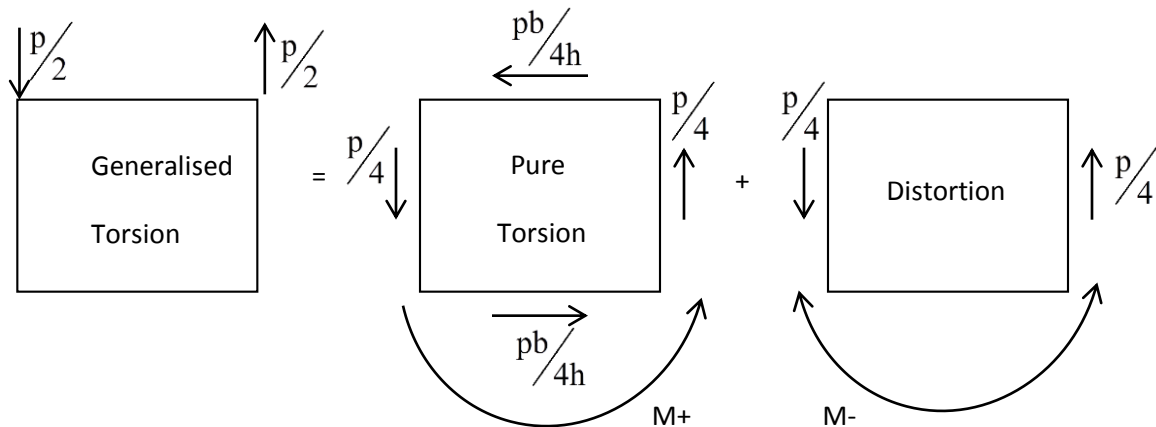


Fig 4.2: Box beam under generalized torsion.

→ From the box beam diagram in fig.4.2, the maximum moment causing the generalized torsion is given as

$$M_t = \frac{P}{2} \times b = \frac{Pb}{2} \dots\dots\dots (4.17)$$

The moment given by Vlasov's as in Osadebe N.N and Mbajogu M.S.W; (2005). is given as

$$M_{(x,s)} = V_{(x)} M_{(s)} \dots\dots\dots (4.18)$$

Where M= The bending moment generated in the cross sectional frame of unit width, when there is a unit distortion V(s) = 1.

The total potential energy on the unit cell due to the distortional load of intensity of is given by the combine effects of

- (a) Due to direct stresses,
- (b) Due to shear stresses
- (c) Due to moment – Twisting
- (d) Due to external loading

In summary the required direct stress equations are

$$\sigma(x, s) = E_x \left( U_{(x)}^1 \phi_{(s)} + V_{(x)}^1 \phi_{(s)} \right) \text{Longitudinal warping stress} \dots\dots\dots (4.15a)$$

$$\sigma(y, s) = E_y \left( U_{(y)}^1 \phi_{(s)} + V_{(y)}^1 \phi_{(s)} \right) \text{Transverse warping stress} \dots\dots\dots (4.15b)$$

Also for shearing stresses, we obtain

$$\tau_{(x,s)} = G \left[ U_{(x)}^1 \phi_{(s)} + V_{(x)}^1 \psi_{(s)} \right] G \dots \text{Longitudinal shearing stress} \dots\dots\dots (4.16a)$$

$$\tau_{(y,s)} = G \left[ U_{(y)}^1 \phi_{(s)} + V_{(y)}^1 \psi_{(s)} \right] G \dots \text{Transverse shearing stress} \dots\dots\dots (4.16b)$$

The general potential energy of the timber box structure under the action of a distortional load of intensity P is given by

$$\pi = \frac{1}{2} \left[ \int_0^l \int_{(s)} \left[ \frac{\sigma^2(x, s)}{E_x} + \frac{\sigma^2(y, s)}{E_y} + \frac{\tau^2(x, s) + \tau^2(y, s)}{G} \right] t_{(s)} + \frac{M^2(x, s)}{E_x I_{(s)}} + \frac{M^2(y, s)}{E_y I} - 2qV_{(x,s)} \right] dx ds \dots\dots\dots (4.19)$$

Where  $I_{(s)} = \frac{t_{(s)}^3}{12(1-\gamma^2)}$  is the second area moment of the cross-sectional frame wall thickness t(s).

Substituting equations (4.15a; 4.15b; 4.16a; 4.16b) into equation(4.19)

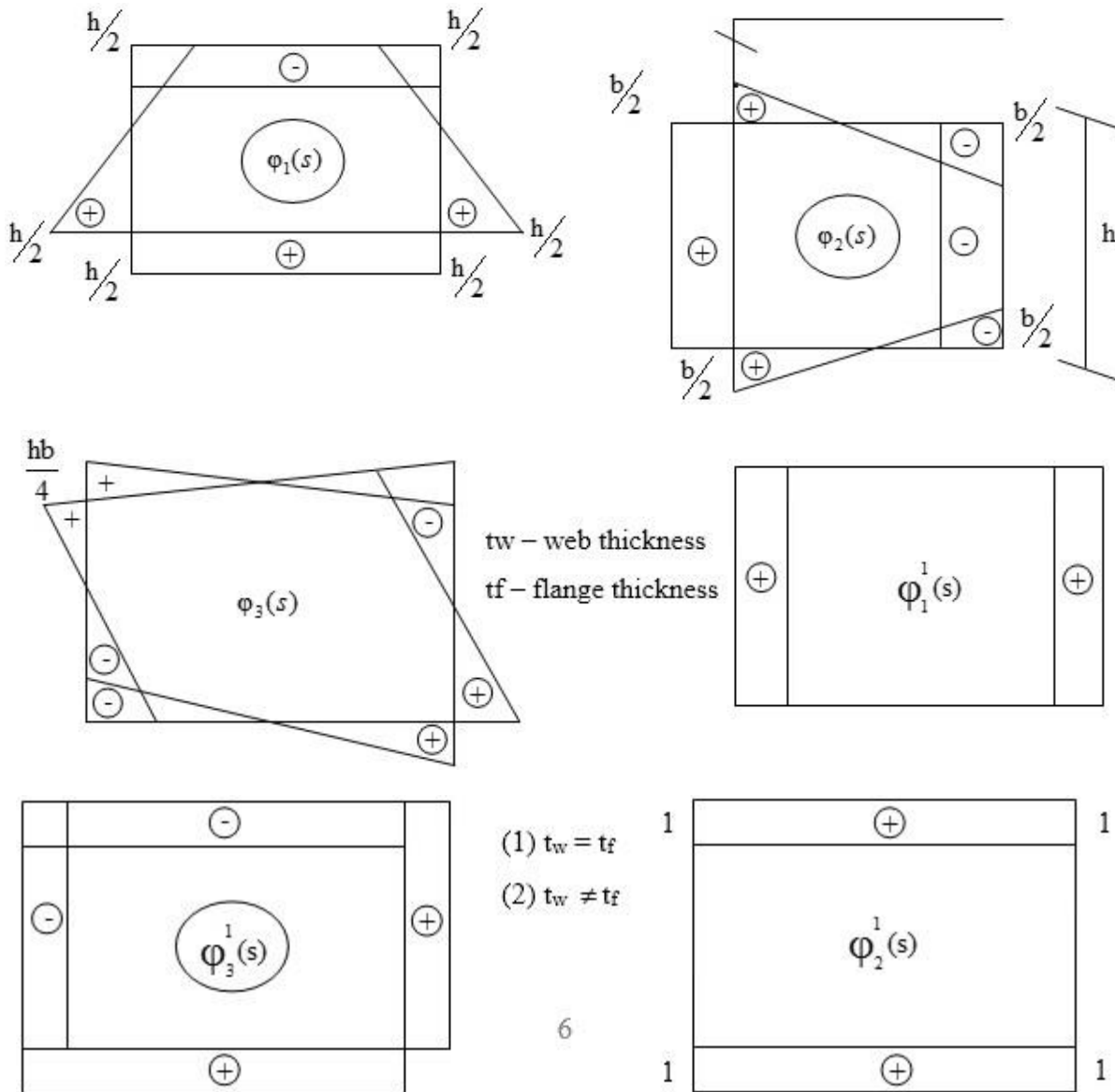
$$\pi = \frac{1}{2} \left[ \int_0^l \int_{(s)} \left[ \frac{E_x^2 \left[ U_{(x)}^1 \phi_{(s)} + V_{(x)}^1 \phi_{(s)} \right]^2}{E_x} + E_y^2 \frac{\left( U_{(y)}^1 \phi_{(s)} + V_{(y)}^1 \phi_{(s)} \right)^2}{E_y} + G^2 \frac{\left[ U_{(x)}^1 \phi_{(s)} + V_{(x)}^1 \psi_{(s)} \right]^2}{G} + G^2 \frac{\left[ U_{(y)}^1 \phi_{(s)} + V_{(y)}^1 \psi_{(s)} \right]^2}{G} + \frac{M_{(x,s)}^2}{E_{(s)} I_{(s)}} + \frac{M_{(y,s)}^2}{E_{(s)} I_{(s)}} - 2qV_{(x,s)} \right] dx ds \dots\dots\dots (4.20)$$

Expanding the above equation gives the generalized potential energy equation as shown below.

$$\pi = \frac{1}{2} \int_0^t \int_s \left[ E_x \left( U_{(x)}'^2 \phi_{(s)}^2 + 2U_{(x)}' V_{(x)}' \phi_{(s)}^2 + V_{(x)}'^2 \phi_{(s)}^2 \right) + E_y \left( U_{(y)}'^2 \phi_{(s)}^2 + 2U_{(y)}' V_{(y)}' \phi_{(s)}^2 + V_{(y)}'^2 \phi_{(s)}^2 \right) + G \left( U_{(x)}^2 \phi_{(s)}'^2 + 2U_{(x)} V_{(x)}' \phi_{(s)}' \psi_{(s)} + V_{(y)}'^2 \psi_{(s)}^2 \right) + G \left( U_{(y)}^2 \phi_{(s)}'^2 + 2U_{(y)} V_{(y)}' \phi_{(s)}' \psi_{(s)} + V_{(y)}'^2 \psi_{(s)}^2 \right) \right] dxs + \frac{M_{(x,s)}^2}{E_{(s)} I_{(s)}} + \frac{M_{(y,s)}^2}{E_{(s)} I_{(s)}} - 2q V_{(x,s)} \dots \dots \dots (4.21)$$

In consideration of various expected strain modes of box beam multiplication diagram, the various strain modes are given as Fig 4.3

**STRAIN MODES**



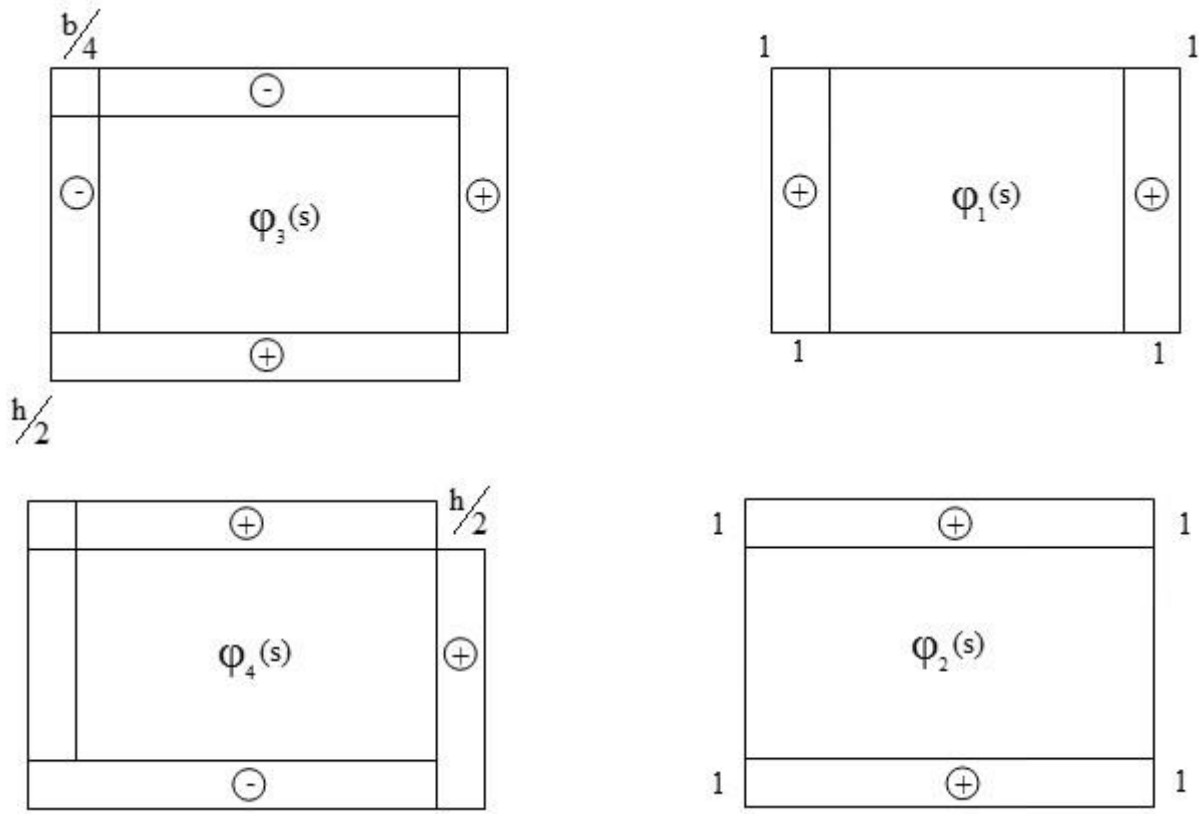


Fig 4.3: Strain modes of box beams

From the diagram multiplication function, the choices of strain mode (shape function) are as follows:

$$\begin{aligned}
 I_1 &= \int_{(s)} \varphi_{(s)}^2 t_{(s)} ds & ; & \quad C_x = \int_0 \frac{\mathbf{M}_{(s,s)}^2}{E_{(s)} I_{(s)}} . ds \\
 I_2 &= \int_{(s)} \varphi_{(s)}^2 t_{(s)} ds & ; & \quad C_y = \int_0 \frac{\mathbf{M}_{(y,s)}^2}{E_{(s)} I_{(s)}} . ds \\
 I_3 &= \varphi_{(s)}^2 \psi_{(s)} t_{(s)} ds & ; & \quad \bar{q}(x) = \int_{(s)} q(x) V_{(x,s)} ds \\
 I_4 &= \varphi_{(s)}^2 t_{(s)} ds ; . & & \quad \dots\dots\dots(4.22)
 \end{aligned}$$

In substituting equations (4.22) in the eqn. (4.23)

$$\pi = \frac{1}{2} \int_0^t \left[ \begin{aligned} & E_x \left[ U_{(x)}'^2 I_1 + 2 U_{(x)}' V_{(x)}' I_1 + V_{(x)}'^2 I_1 + E_y \left[ U_{(y)}'^2 I_1 + 2 U_{(y)}' I + V_{(y)}'^2 I_1 \right. \right. \\ & + G \left( U_{(x)}^2 I_2 + 2 U_{(x)} V_{(x)}' I_3 + V_{(x)}'^2 I_4 + G U_{(y)}^2 I_2 + 2 U_{(y)} V_{(y)}' I_3 + V_{(y)}'^2 I_4 \right) \\ & \left. \left. C_x + C_y - 2q(x) \right] \dots\dots(4.24) \right. \end{aligned} \right.$$

Simplifying further, we obtain that

$$\pi = \frac{1}{2} \int_0^t \left[ E_x I_1 \left[ U_{(x)}'^2 + 2 U_{(x)}' V_{(x)}' + V_{(x)}'^2 \right] + E_y I_1 \left[ U_{(y)}'^2 + 2 U_{(y)}' V_{(y)}' + V_{(y)}'^2 \right] \right. \\ \left. + G I_2 \left[ U_{(x)}^2 + U_{(y)}^2 \right] + 2 G I_3 \left( U_{(x)} V_{(x)}' + U_{(y)} V_{(y)}' \right) + G I_4 \left( V_{(x)}'^2 + V_{(y)}'^2 \right) + C_x + C_y - 2 q_{(x)} \right] \dots(4.25)$$

Equations of distortional equilibrium are obtained by minimizing the above function with respect to its functional variables U and V using the well known Euler-lag range theorem.

Thus the above equation can be minimized using the three equations below.

$$\frac{d\pi}{du} - \frac{d}{dx} \left( \frac{d\pi}{du} \right) = 0 \dots\dots\dots(4.26a)$$

$$\frac{d\pi}{dv_x} - \frac{\partial}{\partial x} \left( \frac{\partial \pi}{\partial v_x'} \right) = 0 \dots\dots\dots(4.26b)$$

$$\frac{\partial \pi}{\partial v_y} - \frac{d}{dx} \left( \frac{\partial \pi}{\partial v_y'} \right) = 0 \dots\dots\dots(4.26c)$$

Considering equation 4.26a we obtain that

$$\frac{\partial \pi}{\partial u} = E_x I_1 \left[ 2 U_{(x)}' + 2 V_{(x)}' + V_{(x)}'^2 \right] + E_y I_1 \left[ 2 U_{(y)}' + 2 V_{(y)}' + V_{(y)}'^2 \right] + G I_2 \left[ 2 U_{(x)} + 2 U_{(y)} \right] + 2 G I_3 \left( V_{(x)}' + V_{(y)}' \right) \\ + G I_4 \left( V_{(x)}'^2 + V_{(y)}'^2 \right) + C_x + C_y - 2 q_{(x)} \dots\dots\dots(4.27)$$

$$\frac{\partial}{\partial x} \frac{\partial \pi}{\partial u'} = E_x I_1 \left[ 2 U_{(x)}'' + 2 V_{(x)}'' + V_{(x)}'^2 \right] + E_y I_1 \left[ 2 U_{(y)}'' + 2 V_{(y)}'' + V_{(y)}'^2 \right] + G I_2 \left( 2 U_{(x)}' + 2 U_{(y)}' \right) \\ + 2 G I_3 \left( V_{(x)}' + V_{(y)}' \right) + G I_4 \left( V_{(x)}'^2 + V_{(y)}'^2 \right) + C_x + C_y - 2 q_{(x)} \dots\dots\dots(4.28)$$

substituting eqns (4.27 and 4.28) into eqn (4.26a),we obtain that

$$\frac{\partial \pi}{\partial u} - \frac{\partial \partial \pi}{\partial x \partial u'} = E_x I_1 \left[ 2 U_{(x)}' - 2 U_{(x)}'' \right] + E_y I_1 \left[ 2 U_{(y)}' - 2 U_{(y)}'' \right] \dots\dots\dots(4.29)$$

Considering equation (4.26b)  $\frac{\partial \pi}{\partial v_x} - \frac{\partial \partial \pi}{\partial x \partial v_x'} = 0$  .we obtain,



$$\frac{\partial \pi}{\partial v_x} = E_x I_1 \left[ U_{(x)}'^2 + 2U_{(x)}' V_{(x)}' + 2V_{(x)}'^2 \right] + E_y I_1 \left[ U_{(y)}'^2 + 2U_{(y)}' V_{(y)}' + V_{(y)}'^2 \right] + GI_2 \left[ U_{(x)}^2 + U_{(y)}^2 \right] + 2GI_3 \left( U_{(x)} + U_{(y)} V_{(y)}' \right) + GI_4 \left( 2V_{(x)}' + V_{(y)}'^2 \right) + C_x + C_y - 2q_{(x)} \dots \dots \dots (4.30)$$

also,

$$\frac{\partial}{\partial x} \frac{\partial \pi}{\partial v_x} = E_x I_1 \left[ U_{(x)}'^2 + 2U_{(x)}' V_{(x)}'' + 2V_{(x)}''^2 \right] + E_y I_1 \left[ U_{(y)}'^2 + 2U_{(y)}' V_{(y)}'' + V_{(y)}''^2 \right] + GI_2 \left[ U_{(x)}^2 + U_{(y)}^2 \right] + 2GI_3 \left( U_{(x)} + U_{(y)} V_{(y)}' \right) + GI_4 \left( 2V_{(x)}' + V_{(y)}'^2 \right) + C_x + C_y - 2q_{(x)} \dots \dots \dots (4.31)$$

substituting eqn(4.30 and 4.31) into eqn(4.26b)

we have that

$$\frac{\partial \pi}{\partial v_x} - \frac{\partial}{\partial x} \frac{\partial \pi}{\partial v_x} = E_x I_1 \left[ 2V_{(x)}' - 2V_{(x)}'' \right] + GI_4 \left[ 2V_{(x)}' - 2V_{(x)}'' \right] \dots \dots \dots (4.32)$$

Considering the last equation to be optimized, we have:

$$\frac{\partial \pi}{\partial v_y} = E_x I_1 \left[ U_{(x)}^2 + 2U_{(x)}' V_{(x)}' + V_{(x)}'^2 \right] + E_y I_1 \left[ U_{(y)}'^2 + 2U_{(y)}' V_{(y)}' + 2V_{(y)}'^2 \right] + GI_2 \left[ U_{(x)}^2 + U_{(y)}^2 \right] + 2GI_3 \left( U_{(x)} V_{(x)}' + U_{(y)} \right) + GI_4 \left( V_{(x)}'^2 + 2V_{(y)}' \right) + C_x + C_y - 2q \dots \dots \dots (4.33)$$

also.

$$\frac{\partial}{\partial x} \frac{\partial \pi}{\partial v_y} = E_x I_1 \left( U_{(x)}'^2 + 2U_{(x)}' V_{(x)}'' + V_{(x)}''^2 \right) + E_y I_1 \left( U_{(y)}'^2 + 2U_{(y)}' V_{(y)}'' + 2V_{(y)}''^2 \right) + GI_2 \left( U_{(x)}^2 + U_{(y)}^2 \right) + 2GI_3 \left( U_{(x)} V_{(x)}' + U_{(y)} \right) + GI_4 \left( V_{(x)}'^2 + 2V_{(y)}'' \right) + C_x + C_y - 2q_{(x)} \dots \dots \dots (4.34)$$

substituting eqn(4.33 and 4.34) into eqn(4.26c) we have

$$\frac{\partial \pi}{\partial v_y} - \frac{\partial}{\partial x} \frac{\partial \pi}{\partial v_y} = E_y I_1 \left[ 2V_{(y)}' - 2V_{(y)}'' \right] + \left[ 2V_{(y)}' - 2V_{(y)}'' \right] \dots \dots \dots (4.35)$$

The summary of potential energy optimized equations of equilibrium are obtained as

$$\frac{\partial \pi}{\partial u} - \frac{\partial}{\partial x} \frac{\partial \pi}{\partial u^1} = E_x I_1 [2 U'_x - 2 V''_{(x)}] + E_y I_1 [2 U''_y - 2 V''_{(y)}] \text{-----} (4.29)$$

$$\frac{\partial \pi}{\partial v_x} - \frac{\partial}{\partial x} \frac{\partial \pi}{\partial v_x^1} = E_x I_1 [2 V'_{(x)} - 2 V''_{(x)}] + G I_4 [2 V'_{(x)} - 2 V''_{(x)}] \text{-----} (4.32)$$

$$\frac{\partial \pi}{\partial v_y} - \frac{\partial}{\partial x} \frac{\partial \pi}{\partial v_y^1} = E_y I_1 [2 V'_{(y)} - 2 V''_{(y)}] + G I_4 [2 V'_{(y)} - 2 V''_{(y)}] \text{-----} (4.35)$$

let  $A = E_x I_1$ ,

$B = E_y I_1$ ,  $C = G I_4$ ,

$$\therefore A [2 U'_{(x)} - 2 V''_{(x)}] + B [2 U'_y - 2 V''_{(y)}] = 0 \text{-----} (4.29)$$

$$A [2 V'_{(x)} - 2 V''_{(x)}] + C [2 V'_{(x)} - 2 V''_{(x)}] = 0 \text{-----} (4.32)$$

$$B [2 V'_{(y)} - 2 V''_{(y)}] + C [2 V'_{(y)} - 2 V''_{(y)}] = 0 \text{-----} (4.35)$$

$$\text{from 4.32} \Rightarrow A + C = 0 \quad = A = -C$$

$$B + C = 0 \quad B = -C$$

$$\therefore \text{from 4.29} = \frac{A}{B} = \frac{2 U'_{(x)} - 2 U''_{(x)}}{2 U'_{(y)} - 2 U''_{(y)}} = \frac{E_x I_1}{E_y I_1}$$

**DISCUSSION:**

1. From the formulated Vlasov's theorem for anisotropic timber box beam. The above equations therefore show second derivatives distortional function of box beam with both the longitudinal and the transverse distortional component. It shows clearly that the answer lies in serviceability not in the ultimate load.
2. It has reduced the rigorously method of design to a simple test of bending test in timber; as a result makes it easier for engineers to work on.
3. It has remove the destructive test of any existing trusses (in-service) which will posses serious danger to the lives of inhabitants and the structural frame already in service.
4. It obviates the series of mode of vibrations of test rigs, possible accidents due to sudden catastrophic collapse of the trusses and the rigs.
5. It has reduced the cost/risk associated with the acquisition hire of crane, pay loader and construction of test rigs. Therefore for a bending of constant (EI) and of any span, the load factors method by the Vlasov's theorem applies.
6. The formulated energy equation will be used in analysis of the stress problem, and the generalized torsion in anisotropic materials (timber).

**CONCLUSION:**

Vlasov's energy equations for generalized torsion of anisotropic material (timber) have been developed. In three equations obtained, the first equation shows a coupled equation of the generalized distortion in both longitudinal direction and transverse direction why the other two shows the particular directional distortion. Also the formulated distortions appear less than the measured distortion and this is in correlations with the specification on deflection limit and thus gives a good index of the distortion in the generalized distortion. This will be useful in predicting the distortional and warping torsion of timber box beams.

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